Chapter 9. On calculation of elementary particles' masses

1.0. Introduction.

1.1. Equivalence of the energy and mass spectra

As it is known neither classical, nor quantum theories could explain the nature of masses of elementary particles and could not deduce the numerical values of masses till now.

The basic experimental facts are here the following: 1) masses of elementary particles make the discrete spectra; 2) all elementary particles are the excited states of a small set of some particles, which represent the lowest level of a spectrum of masses.

It is supposed that discreteness of spectrum of masses of elementary particles is similar to a discrete spectrum of excitation energies of atom. According to the Einstein formula $\varepsilon = mc^2$, to any rest mass corresponds the stationary level of energy.

1.2. Energy spectra of electron in hydrogen atom as an example of a spectrum of masses

The first calculation of energy-mass spectrum of electron in hydrogen atom has been based on the known Bohr atom theory, in which the quantization was entered by a separate postulate. This approach allows the calculation of energy spectrum of electron, but it does not reveal the reasons of quantization.

The reasons of quantization have been specified by de Broglie, who showed that elementary particles in a stationary state can be considered as standing waves, which formation conditions are the conditions of the length waves' integrality.

In his dissertation de Broglie has shown (Broglie, de, 1924; 1925;. Andrade e Silva and Loshak, 1972), that the orbits postulated by Bohr for electron motion around a hydrogen atom nucleus can be received from the condition that the length of an orbit L should contain an integer number of electron wavelengths $L = n\lambda$ (where $\lambda = h/m\upsilon = 2\pi\hbar/p$ is the particle wavelength according to de Broglie, $h, \hbar$ is the Planck constant (usual and bar), υ is the particle velocity, $p = m\upsilon$ is particle momentum) (see fig. 1):

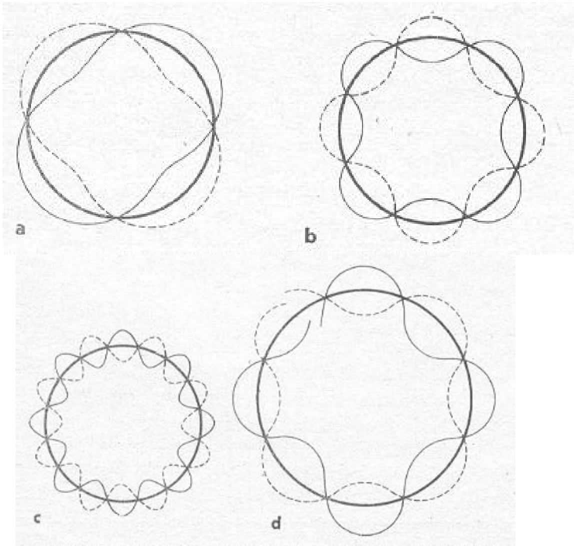

Fig. 1

For **a** , **b** and **c** of fig.1 this condition is carried out, when *n* = 2, 4 and 8, accordingly. In case of **d** this condition is not carried out and electron motion is unstable, that leads to self-destruction of a wave as a result of the wave interference. Mathematically the integrality condition corresponds to the requirement of unambiguity of wave function.

A similar condition also takes place for elliptic orbits (see also (Shpolski, 1951)), but this case is more complex, since the length of de Broglie wave in different points of an elliptic orbit varies because the electron speed is not constant. In this case it is necessary to use the general condition of quantization:

$$\int \frac{ds}{\lambda} = \int_0^T \frac{m\beta^2 c^2}{h\sqrt{1-\beta^2}} dt = n, \qquad (1.1)$$

where ds is the orbit length element, T is the period of motion, dt - time element, $\beta = c/\upsilon$.

From the above follows that the stationarity conditions correspond to resonance conditions, which are adequate to conditions of integrality of the standing waves.

Such sight at the reason of appearance of quantum levels of electron energy also allows to calculate the last in other similar cases. For example, as an approximate model of 3-dimensional short-range potential, can be the spherical potential well of some radius *R* (Naumov, 1984). According to de Broglie for the big circle of sphere of radius *R*, we will have:

$$2\pi R = n\lambda = n\frac{2\pi\hbar}{p} = n\frac{2\pi\hbar}{\sqrt{2m\varepsilon}}, \qquad (1.2)$$

From here, we receive for energy levels: $\varepsilon_n = \frac{\hbar^2 n^2}{2mR^2}$. As we marked above (see chapter 8), the exact solution of this problems as the Helmholtz equation for de Broglie waves (i.e. the Schreudinger wave equation) gives only additional factor π^2.

We should note one remarkable feature, which has the solution of Schreudinger equation for electron in a potential well of final depth. The solution shows (Shiff, 1955; Matveev, 1989), that in this case there is only a limit number of own levels of energy. Whether it is possible to extend this conclusion to elementary particle mass calculation (e.g. to the charge leptons, which consists from three flavors only), remains in doubt.

Attempts of calculation of mass-energy spectra on the basis of resonance behaviour of particles exist for a long time. We will briefly mention the most consecutive of them.

2.0. Present calculations of elementary particle masses

The existing calculations are based on assumptions and guesses, which cannot be proved enough within the framework of the quantum field theory.

2.1. Quasi-classical approaches to mass calculation

According to them the basic particle assimilates to a potential well (or, that is the same, to the resonator of the certain configuration). The spectrum of masses of particles arises, when some additional resonance particle (e.g. photon) is placed in this potential well. Characteristics of addition particle change the characteristics (mass, spin, charge, etc) of the basic particle and we can consider the last one as new particle.

One of the first attempts of quasi-classical calculation (for masses of muon and pion) belongs to K. Putilov (Putilov, 1964). Note that this calculation does not take into account the experimental facts, which have been found out later (e.g., the existence of a tau-lepton, the law of lepton number conservation, etc.) and it should be considered only as an example of a corresponding computational procedure.

A second, much more detailed calculation (for the big number of particles, known at that time) is stated in the paper (Kenny, 1974). Here the author gives already the theoretical substantiation of a calculation method and receive impressing results. But calculation is made by analogy to the theory of Bohr; as a result here Coulomb potential well was used. The obtained numerical values of masses, without serious substantiation, are corresponded with masses of known particles.

Another approach is based on the quantization rules of Bohr-Wilson-Sommerfeld. The group of scientists J.L.Ratis, F.A.Garejev and others (Ratis and

Garejev, 1992; etc) has achieved especially impressive results, using the quantization condition for asymptotic momenta of decay products of the hadronic resonances.

2.2. Quantum approaches to mass calculation

The calculations, based on idea of composite particles, take place here. But we will show below that this approach has near connection to the resonance theory.

As it is marked in the reviews (Rivero and Gsponer, 2005; Gsponer and Hurni, 2005) one of the first possible approaches to an estimation of masses of elementary particles was based on the known composite model of Nambu-Barut (Nambu, 1952; Barut, 1979). In this approach it is postulated that for calculation of masses of heavy leptons to the rest mass of electron the quantized magnetic energy $(3/2)\alpha^{-1}\sum_{n=0}^{n=N} n^4$ must be added, where n is a new quantum number.

In particular, for leptons Barut has received the following empirical formula: $m(N) = m_e\left(1+\frac{3}{2\alpha}\sum_{n=0}^{n=N} n^4\right)$, which gives satisfactory values for both heavy leptons (here m_e is the electron mass, α is electromagnetic constant).

The authors of the paper (Gsponer and Hurni, 2005) write: "The agreement with the data of this rather simple formula is surprisingly good, the discrepancy being of order 10^{-4} for the muon and 10^{-3} for the tau, respectively. In order to get the masses of the quarks, it is enough to take for the mass of the lightest quark $m_u = m_e/7.47$. Again, we see in Table 1 (see below) that the agreement between the theoretical quark masses and the "observed" masses is quite good, especially for the three heavy quarks".

Table 1: Comparison of lepton and quark masses in Mev/c2 calculated with Barut's formula to measured masses.

	Lepton masses			**Quark masses**		
N		Barut's formula	Data		Barut's formula	Data
0	e	0.511	0.511	u	0.068	0-8
1	μ	105.55	105.66	d	14.1	5-15
2	τ	1786.1	1784.1	s	239	100-300
3		10294	?	c	1378	1300-1500
4		37184	?	b	4978	4700-5300
5				t	13766	?
6					31989	?

Recently, an expression, similar to Barut, had been received from other reasons. In the paper (Rodriguez and Vases, 1998) for muon mass as excited state of electron (which is allocated with properties of quark) the formula is received: $m_l = \left(1 + \frac{q_m^n}{e}\right) m_e$, where $q_m^n = \frac{3e}{2\alpha} n$. E.g., for muon at n = 1 turns out: $m_\mu = \left(1 + \frac{3}{2\alpha}\right) m_e = 206{,}55 m_e$. Assuming that taon is the excited state of muon, authors receive also:

$m_l = \left(1 + \frac{3}{2\alpha}\right) m_e + \frac{q_m^n}{e} m_e = \left(1 + \frac{3}{2\alpha}\right) m_e + \frac{3}{2\alpha} n$, that at $n = 16$ gives for taon mass the value close to experimental, namely $m_\tau = 3494 m_e = 1781{,}9$ MeV.

Other successful empirical formula is I. Koide's formula (Rivero and Gsponer, 2005), which was descavered "on the end of year1981. I. Koide, working above some composite model of quarks, has had fortunate or unfortunate case to run into very simple correlation among three charge leptons $\left(m_e + m_\mu + m_\tau\right) = \frac{2}{3}\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2$, which gives for mass of a tau-lepton 1777 MeV ".

Below we will show that there is a successive approach, which doesn't contradict to quantum field theory and allows to obtain strictly enough the formulas, close to the formulas of K.A Putilov, A.O. Barut, W.A Rodrigues.- J. Vaz, and also to confirm the calculation formula of Yu. L. Ratic – F.A. Gareev-et al. (In the framework of this analysis about I. Koide formula we cannot say anything).

3.0. Statement of problems of calculation of masses of particles

Without the contradiction with quantum field theory and according to CWED we can suppose that:

- all particles are divided into two groups: a) absolutely stable particles: electron, neutrino, proton and their antiparticles; b) metastable particles: all other particles.
- the stable elementary particles are the simplest twirled waves
- the metastable elementary particles are the composite (compound) twirled waves, appearing as superposition of an absolutely stable twirled waves and some additive twirled waves.
- the metastability of compound particles is ensured by resonance conditions and by corresponding conservation laws.

As a simplest reaction of a compound particle formation it is possible to consider the transition of electron e^- in hydrogen atom from a low level to higher level of energy:

$$e^-(\varepsilon_n) + N = \gamma + e^-(\varepsilon_b) + N,$$

where $\varepsilon_n > \varepsilon_b$, ε_b is the base electron energy, ε_n is any electron energy level, γ is the photon (gamma-quantum), and N means the nucleus field, which in this case as a resonator works. Note that the reaction of electron-positron pair production from a photon can be described formally in the same way:

$$\gamma + N = e^- + e^+ + N,$$

The above reactions in a general view can be presented as follows:

$$X_3 \Leftrightarrow X_1 + X_2, \qquad (3.1)$$

where the letter *X* stands for particles; the index 3 stands for the compound particle, and 1 and 2 – for the initial particles. For each of particles we can write the energy-momentum conservation law:

$$\varepsilon_1^2 = c^2 p_1^2 + m_1^2 c^4, \qquad (3.2)$$

$$\varepsilon_2^2 = c^2 p_2^2 + m_2^2 c^4, \qquad (3.3)$$

$$\varepsilon_3^2 = c^2 p_3^2 + m_3^2 c^4, \qquad (3.4)$$

where c is the speed of light, m in the this chapter means the particle rest mass; ε and p are the energies and momentums respectively. For the reaction (3.1) the energy and momentum conservation lows are following:

$$\varepsilon_3 = \varepsilon_1 + \varepsilon_2, \qquad (3.5)$$

$$\vec{p}_3 = \vec{p}_1 + \vec{p}_2, \qquad (3.6)$$

3.1. Direct problem

The record (3.1) is possible to be considered as an instruction

$$X_3 + \overline{X}_1 \Leftrightarrow X_2$$

that a particle X_3 (e.g., electron) as the resonator with known parameters absorbs a antiparticle X_1 (e.g., photon) and we must find the parameters of the resulting particle X_2 (we will name this problem as direct problem).

It is easy to understand that the problem of this type is reduced to the solution of the non-linear wave equations of Heisenberg non-linear equation type. Unfortunately, the solution of the last, despite of a number of achievements (in particular, the existence of spectra of masses of particles has really been shown), had difficulties, which are not overcome till now. Therefore we must try to solve below this problem in linear approach, taking into account the known integrated characteristics of an initial particle (in this case, of electron).

3.2."Inverse" problem

Another statement of the problem (an "inverse" problem of particle mass calculation) arises in case (see (3.1)) we consider the particles 1 and 2 as composite parts of particle 3, but the parameters of particle 3 as resonator we don't know.

Let X_3 is a compound particle with unknown mass. In the simplest case of motionless particle we have $\vec{p}_3 = 0$. Then from (3.24 we receive $\varepsilon_3 = m_3 c^2$, and from (3.6): $\vec{p}_1 = -\vec{p}_2$. Entering a designation $|\vec{p}_1| = |\vec{p}_2| = p_r$, from (3.5) with help (3.2-3.3.)we will receive the known kinematics expression:

$$m_3 c^2 = \sqrt{m_1^2 c^4 + c^2 p_r^2} + \sqrt{m_2^2 c^4 + c^2 p_r^2}\,, \tag{3.7}$$

The question is here how to calculate p_r. For the clarification of the last we will use of the de Broglie approach.

The following paragraphs will be devoted to the briefly analysis of kinematics and wave properties of particles, which can useful for the particle mass calculation.

4.0. The kinematics characteristic of particles

The energy, momentum and kinetic energy of the particles are defined by following relativistic expressions:

$$\varepsilon = mc^2\delta\,,\ \vec{p} = m\vec{\upsilon}\delta\,,\ \varepsilon_k = mc^2(\delta - 1), \tag{4.1}$$

where $\delta = 1/\sqrt{1-\beta^2}$, $\beta = \vec{\upsilon}/c$, $\vec{\upsilon}$ is speed of a particle.

Since $\upsilon < c$, the expressions, containing δ, can be expanded to Maclaurin series (taken here into account only 4 terms):

$$\delta = 1 + \left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\}, \tag{4.2}$$

$$\beta\delta = 0 + \beta + 0 + \frac{1}{2}\beta^3 + \ldots \tag{4.3}$$

and we can receive for energy and momentum the following expressions:

$$\varepsilon = mc^2 + mc^2\left\{\frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots\right\}, \tag{4.4}$$

$$\varepsilon_k = \varepsilon - mc^2 = mc^2 \left\{ \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + \frac{35}{128}\beta^8 + \ldots \right\}, \qquad (4.5)$$

$$p = mc\beta + \frac{1}{2}mc\beta^3 + \ldots = m\upsilon + \frac{1}{2}m\frac{\upsilon^3}{c^2} + \ldots, \qquad (4.6)$$

At $\beta << 1$ we obtain from (4.4)-(4.6) as first approximation the classical expressions:

$$\varepsilon \approx mc^2 + \frac{1}{2}m\upsilon^2 = \varepsilon_{cl}; \;\; p \approx m\upsilon = p_{cl}; \; \varepsilon_k \approx \frac{1}{2}m\upsilon^2 = \varepsilon_{kcl}; \qquad (4.7)$$

where the index “*cl*” is for “*classical*”.

So, as we showed, the enumerated above the particle-wave characteristics can be expanded in the convergent exponential series. With the speed of equal to zero we have the first term: particle with the constant mass - rest mass. When particle begins to move, the infinite sum of terms appears. If we take as the limitation of the series the maximally measurable today value of mass and will not examine the contribution of the terms, smaller than this mass, then it is possible to represent the following picture.

The moving particle seemingly consists of the sum of several particles X_i, where $i = 0,1,2,\ldots N$, and N is determined by speed, so that the mass of moving particle is determined by:

$$m(\upsilon) = \varepsilon / c^2 = m\sum_{i=0}^{N} k_i \beta^{2i}, \qquad (4.8)$$

where $k_i < 1$ are the numerical coefficients of the terms of the series.

The value $i = 0$ is related here to the particle in “the rest”, which gives the basic contribution to the value of mass. With the growth of velocity it grows both the number N and the masses of particles with $i \geq 1$. Thus, with the sufficiently high speed, besides initial electron we have at the given point a set of additional particles. It is unintelligible if the appearance of such particles is kinematics effect or it is connected with interaction of particles with the physical vacuum.

The masses of the particle, additional to the rest electron, are considerably less than the rest mass of electron and they can be difficultly measured. For the proton the result is somewhat better. Since in the proton there are three point heavy quarks, the number of additional particles “inside” the proton at the same speed will be many times more than for the lepton.

What these additional particles do present, and are there any experimental data, which confirm this picture? If we speak about the electron, then such data are unknown to us. For the proton some experiments can be interpreted in the desirable sense. Actually, with the enough energy of electron-proton collision, together with the quarks the set of the point particles, which are called partons, is revealed. Is it possible to interpret these results in favor of existence of additional particles, we do not know. But, nevertheless, the use of these "expansions into the particles" gives the additional possibilities of the analysis of the behavior of particles.

Let us examine them now from the point of view of wave mechanics.

5.0. The wave characteristics of elementary particles

According to de Broglie the particle with energy ε and momentum p has, taking into account (4.1)-(4.3), the following frequency and wavelength:

$$\nu = \frac{\varepsilon}{h} = \frac{mc^2}{h} + \frac{\varepsilon_k}{h} = \nu_0 + \nu(\upsilon), \tag{5.1}$$

$$\lambda = \frac{h}{p} = h \Big/ mc\left(\beta + \frac{1}{2}\beta^3 + \ldots\right), \tag{5.2}$$

where $\nu_0 = \dfrac{mc^2}{h}$, $\nu(\upsilon) = \dfrac{mc^2}{h}\left\{\dfrac{1}{2}\beta^2 + \dfrac{3}{8}\beta^4 + \dfrac{5}{16}\beta^6 + \dfrac{35}{128}\beta^8 + \ldots\right\}$.

In the case $\upsilon << c$ we will obtain:

$$\nu = \frac{\varepsilon}{h} \approx \frac{mc^2}{h} + \frac{1}{2}m\upsilon^2 \Big/ h, \tag{5.3}$$

$$\lambda = \frac{h}{p} \approx \frac{h}{m\upsilon}, \tag{5.4}$$

Let's analyse these expressions.

At first, we can note the interesting feature of de Broglie wave: it consists of infinite series of waves, which frequencies sum up arithmetically.

Secondly, the "base" wave exists, which high frequency ν_0 does not depend on the particle motion.

Thirdly, a number of waves exist here, the frequencies of which depend on the velocity of the particle motion and correspond to separate terms of expansion $\nu(\upsilon)$. The values of the frequencies of these waves are far less than the frequency ν_0 of the main wave, so that these waves can modulate in some way the main wave. What role do these waves separately play in nature, we do not know, but their sum defines all wave effects of particles motion: motion of electron in the atom, diffraction of electron into slots and other.

Fourthly, the frequency of the "based" wave ν_0 of the particle defines the Compton wavelength of rest particle and its "bare" mass: $\lambda_e = \frac{\hbar}{m_e c} = \frac{1}{c\nu_0}$.

Fifthly, at zero speed of particle motion the de Broglie wave *length* is equal to infinity, while for classical oscillator it corresponds to zero frequency of oscillation. But in this case for de Broglie wave *frequency* we obtain non-zero value ν_0 of the order of 10^{15} Hz, which does not depend on the speed of electron motion.

It is possible to tell that in this case, nature has thought up the smart mechanism: to the big and constant frequency of own wave of the rest electron it adds the frequency of an additional wave, which changes from zero to infinity, depending on the speed of electron motion.

As it is known the de Broglie wave is a clearly relativistic effect, connected with relative motion of electron in relation to other bodies (in particular, to a proton in the atoms and accelerators). As the de Broglie analysis shows, the wave appearance can be connected with relativistic Doppler effect, but the deep reasons of this phenomenon remain unknown for us.

6.0. To calculation of mass spectra of elementary particles

6.1. The direct problem

As examples of the elementary reactions of production and disintegration of elementary particles (see (Review of Particle Properties, 1994).) are:

1) reaction of electron-positron pair production $\gamma + N = e^- + e^+ + N$;

2) muon decay $\mu^\pm = e^\pm + \nu + \overline{\nu}(99\%), \ = e^\pm + \nu + \overline{\nu} + \gamma(1\%)$, and taon decay $\tau^\pm = \mu^\pm + \nu_\tau + \overline{\nu}_\mu(17,37\%), \ \mu^\pm + \nu_\tau + \overline{\nu}_\mu + \gamma(3,6\%)$,

where

$$\begin{gathered} m_\mu = 105,6MeV, \ m_\tau = 1777MeV, \ m_e = 0,51MeV \\ m_{\nu_e} < 3ev, \ m_{\nu_\mu} < 0,19MeV, \ m_\nu \approx 1eV \end{gathered}, \qquad (6.1)$$

We can consider these reactions as superposition of the twirled photons and semi-photons. . In this case muon or taon are possibly thought of as consisting of the electronic linear polarized half-wave and two neutrino circularly polarized half-waves with the opposite direction of rotation, which modulate the first.

It is also similarly possible to consider other reactions, without the infringement of corresponding conservation laws; for example:

3) pions decay: $\pi^{\pm} = \mu^{\pm} + \nu_{\mu}(99{,}98\%)$, $\pi^{0} = 2\gamma(98{,}79\%)$; $\pi^{0} = e^{+} + e^{-} + \gamma(1{,}19\%)$,where $m_{\pi^0} = 134{,}97 MeV$, $m_{\pi^{\pm}} = 139{,}57 MeV$

We will consider a particle X_3 (see (3.1)) as the given resonator, and particles X_1, X_2 as the unknown waves, which satisfy to resonance conditions of this resonator.

Since the unique particle, about the sizes of which we can speak with some share of confidence, is the electron, we will consider above three reactions, taking that the electron is here the lowest level of a mass spectrum. (Here instead of one photon (with spin one in Putilov approach, we have both neutrino and antineutrino with the half spin, moving to the opposite directions; therefore for simplification of calculation of mass we will consider two neutrino as one photon).

As we have shown (see chapter 2) the electron equation can be considered not only as the quantum Dirac equation, but also in non-linear electromagnetic form as the equation of twirled semi-photon. Using this fact, let's consider the Dirac electron equation with an external field:

$$\left[\hat{\alpha}_0\left(\hat{\varepsilon} - \varepsilon_{ph}\right) + c\hat{\vec{\alpha}}\left(\hat{\vec{p}} - \vec{p}_{ph}\right) + \hat{\beta} mc^2\right]\psi = 0, \qquad (6.2)$$

We can group here the mass-energy part as following:

$$\left\{\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \left[\left(\hat{\alpha}_o\varepsilon_{ph} + c\hat{\vec{\alpha}}\,\vec{p}_{ph}\right) + \hat{\beta}\, mc^2\right]\right\}\psi = 0, \qquad (6.3)$$

As the free Dirac electron equation is satisfied by any mass, we can write:

$$m(n) = \left[\left(\hat{\alpha}_o\varepsilon_{ph} + c\hat{\vec{\alpha}}\,\vec{p}_{ph}\right) - \hat{\beta}\, mc^2\right] = \hat{\beta}(m_e + m_{ad})\, c^2, \qquad (6.4)$$

where $m_{ad} = m_{ad}(n)$ is the additional mass, accepting a discrete number of values depending on $n = 1,2,3,\dots$, so that from (6.3) we obtain:

$$\left[\left(\hat{\alpha}_o\hat{\varepsilon} + c\hat{\vec{\alpha}}\,\hat{\vec{p}}\right) - \hat{\beta}(m_e + m_{ad})\, c^2\right]\psi = 0, \qquad (6.5)$$

or, taking into account $\hat{\varepsilon} = i\hbar\,\partial/\partial t$, $\hat{\vec{p}} = -i\hbar\vec{\nabla}$, we can receive:

$$\left[\left(\hat{\alpha}_o\frac{\partial}{\partial t} - c\hat{\vec{\alpha}}\,\vec{\nabla}\right) + i\hat{\beta}\, c\frac{(m_e + m_{ad})c}{\hbar}\right]\psi = 0, \qquad (6.5')$$

The equation (6.5), because of the term of interaction, in the general case is the non-linear equation of the twirled waves. Its solution generally is not yet found. Therefore we will simplify the problem, using resonance conditions.

It is possible to present the mass term in (6.5) (without coefficient $i\hat{\beta}\, c$) as follows:

$$\frac{(m_e + m_{ad})c}{\hbar} = \frac{m_e c}{\hbar} + \frac{m_{ad} c}{\hbar} = \frac{1}{\lambdabar_e} + \frac{1}{\lambdabar_{ad}}, \qquad (6.6)$$

where $\lambdabar_e$, $\lambdabar_{ad}$ are both the Compton waves' lengths (bar) of the electron and of the additional mass, accordingly (where by definition $\lambdabar_C = \hbar/mc = \lambda_C/2\pi mc$; note that the value $\lambda_C = h/mc$ also is referred to as Compton wave length). Since the basic wave contains an integer number of the additional waves (i.e. the basic wave and additional waves should be commensurable), they should satisfy the following condition of wave quantization:

$$\lambda_{ad} = \kappa\frac{\lambda_e}{n} \text{ or } \lambdabar_{ad} = \kappa\frac{\lambdabar_e}{n}, \qquad (6.7)$$

where κ is the number, describing a condition of appearance of a resonance (longitudinal, cross-sectional resonance, etc.); $n = 1,2,3,...$ is an integer (quantum number). In case of propagation of a wave along the circle (as in the above problem (1.2)) we have $\kappa = 2\pi$. In case of wave propagation along the sphere radius $\kappa = 4$, along the cylinder radius $\kappa = 2$. It is possible to assume, that generally in various configurations of particles and fields the constant can also accept other values.

Thus, for mass term in Dirac equation (i.e. for mass of a complex elementary particle) we receive:

$$\frac{c}{\hbar}m_{ep} = \frac{(m_e + m_{ad})c}{\hbar} = \frac{1}{\lambdabar_e} + \frac{1}{\lambdabar_{ad}} = \frac{1}{\lambdabar_e}\left(1 + \frac{\lambdabar_e}{\lambdabar_{ad}}\right), \qquad (6.8)$$

Since the value $\alpha = e^2/\hbar c = r_0/\lambdabar_e \approx 1/137$ represents an electromagnetic constant, we have $\lambdabar_e = r_0/\alpha$ (where $r_0 = e^2/m_e c^2$ is the classical electron radius). Taking this into account, from (6.8) we will receive:

$$\frac{c}{\hbar}m_{ep} = \frac{1}{\lambdabar_e}\left(1 + \frac{\lambdabar_e}{\lambdabar_{ad}}\right) = \frac{\alpha}{r_0}\left(1 + \frac{r_0}{\alpha\lambdabar_{ad}}\right), \qquad (6.9)$$

As we have shown (Kyriakos, 2004a), the "bare" size of electron corresponds to Compton wave length and at polarization in physical vacuum, decreases in $1/\alpha \approx 137$ times. Thus, taking into account the polarization of vacuum, instead of (6.7), we should write down:

$$\lambda_{ad} = \kappa \frac{r_0}{n} \text{ или or } \lambdabar_{ad} = \kappa \frac{r_0}{2\pi n}, \tag{6.10}$$

From here $\dfrac{r_0}{\lambdabar_{ad}} = \dfrac{2\pi}{\kappa} n$, which by substitution in the formula (6.9), gives the formula for mass of a compound particle:

$$m_{ep} = \left(1 + \frac{2\pi}{\kappa\alpha} n\right) m_e \approx \left(1 + \frac{2\pi}{\kappa} 137 \cdot n\right) m_e, \tag{6.11}$$

Using the known constant we obtain from (6.11) following:
- for $n = 0$ we receive a trivial case of electron mass $m_{ep} = m_e$;
- for $\kappa = 4,\ n = 1$: $m_{ep} = 110{,}2$ MeV (that corresponds $m_\mu = 105{,}6$ MeV);
- for $\kappa = 4,\ n = 16$: $m_{ep} = 1755$ MeV (that corresponds $m_\tau = 1777$ MeV);
- for $\kappa = \pi,\ n = 1$: $m_{ep} = 140{,}25$ MeV (that corresponds $m_{\pi^\pm} = 139{,}57$ MeV).

These results are close to the results received from K. Putilov (Putilov, 1964) and from other aforementioned authors.

Although the sequence of a theoretical conclusion of the mass formula makes the casual concurrence improbable, however it would not be necessary to make hasty conclusions.

Let's consider now the results of particle masses' calculation according to the inverse problem.

6.2. Inverse problem.

We will consider here a particle X_3 of the reaction (3.1) as the unknown resonator, and particles $X_i = \{X_1, X_2, ...\}$ as the initial waves, which can select, proceeding from known excited states of this resonator.

In each resonator there are no more than three sizes L_j ($j = 1,2,3$), which define lengths of resonance waves. In conformity with requirements of appearance of standing waves in the resonator, we can write down, at least, three resonance conditions:

$$L_j / \lambda_i = \kappa n_{ij}, \tag{6.12}$$

where i is number of a particle defining, participating in synthesis (in our example $i = 1,\ 2$), $n_{ij} = 1,2,3,...$ is an integer, κ_j is the constant dimensionless coefficient, defining resonance conditions. Since according to de Broglie $\lambda_i = h/p_i$, the formula (6.12) can be rewritten in the form of:

$$p_{ij} \cdot L_j = \kappa_j h n_{ij}, \qquad (6.13)$$

From here at $n_{ij} = 1$ we receive the following condition for lowest states of a particle X_3:

$$p_{ij3} = \kappa_j \cdot h/L_j = const, \qquad (6.14)$$

Then for any other "excited" state of a particle X_3 we have:

$$p_{ij} = \kappa_j \frac{h}{L_j} n_{ij} = p_{ij3} \cdot n_{ij}, \qquad (6.15)$$

In case of two fusion particles ($i = 2$) we have for lowest momentums and quantum numbers values, which depend only on the index j (namely $p_{1j} = p_{2j} = p_{j3}$) and on integers n_j. Thus, knowing only one number for the given resonance, we can calculate by (3.7) the masses for different j.

In works of group of Ratis, Yu.L., Garejev, F.A. et all (Ratis and Garejev, 1992; Garejev, Kazacha et al, 1998;, etc.) values p_{j3} for the big group of hadron resonances, which give encouraging acknowledgement to our calculations, were selected.

In case if the number of particles X_i is more than 2 (i.e. at $i > 2$) it is necessary for the calculation of spectra to have additional correlations between p_{ij} for different i.

As the samples, we present from the paper (Garejev, Kazacha, etc., 1998) the results of calculation of several resonances (see below the appendix). (Note that the paper (Garejev, Barabanov, etc., 1997) contains the analysis of some hundreds resonances, corresponding to the above conditions).

The appendix (the data is taken from (Garejev, Kazacha, etc., 1998)):

Table 1. Invariant masses of the resonances which are decay along binary channels with momentums, which are multiple to 29,7918 MeV/c: Pn = n x 29,7918 MeV/c

Resonanses	Decay chanels	P_{exp}	n	P_{exp}/n	M_{exp}	M_{th}	ΔM
$\pi^{\pm}$	$\mu^{\pm}\nu_{\mu}$	29,79	1	29,79	139,56	139,56	--
ρ (770)	$\pi^{\pm}\pi^{\mp}$	358	12	29,83	768,5	767,56	0,94
f_2(1810)	$\pi^{\pm}\pi^{\mp}$	896,70	30	29,89	1815	1809,17	2,17
ρ_5 (2350)	$p\overline{p}$	714,75	24	29,87	2359	2359,31	0,31
X(2850)	$p\overline{K}^0$	171,08	6	28,51	2850	2850,0	0,0

Table 2. Invariant masses of the resonances, which decay along binary channels with momentums, which are multiple to 26,1299 MeV/c: Pn=n x 26,1299 MeV/c

Resonanses	Decay chanels	P_{exp}	n	P_{exp}/n	M_{exp}	M_{th}	ΔM
π^0	$\mu^{\pm}e^{\mp}$	26,12	1	26,12	134,97	134,97	--
D^0	$\overline{K}^0 f_0$ (980)	549	21	26,14	1864,5	1863,97	0,53
$D^{\pm}$	$\overline{K}^0\pi^{\pm}$	862	33	26,12	1869,3	1869,11	0,19
Λ_c^+	$\Xi(1530)^0 K^+$	471	18	26,17	2284,9	2284,25	0,65
B^0	e^+e^-	2639	101	26,13	5279,2	5278,24	0,96

Unfortunately, the volume of the paper does not allow to consider other results of CWED and to give interpretation of many of SM results from the point of view of CWED.